

# Mathematical Description of the Regensburg Model Scenario Types RM 1 – 6

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## 1 Introduction

The Regensburg Model Scenario Types are used to derive plausible emission paths that meet a certain budget.

The emission paths are essentially determined indirectly by an assumption about the property of the annual changes. This is the innovative core of the RM Scenario Types. The scenario types RM 1 – 5 represent typical monotonous annual reduction rates. RM-6 assumes a constant annual reduction amount.

Our goal is to discuss the following three basic types of **courses of reduction rates**:

- (1) Initial less than proportional increase<sup>1</sup> (RM-2 and RM-4) ► concave
- (2) Initial over-proportional increase (RM-5) ► convex
- (3) Linear increase (RM-3) ► linear

In principle, there are several options for mapping the basic types (1) and (2) using a specific function. However, as the scenario types RM-2 and RM-4 show, the results usually do not differ significantly with a tight budget and a plausible course of the reduction rates.

RM-1 with a constant reduction rate and RM-6 with a constant reduction amount serve primarily as key figures for the size of the challenge. They are less suitable because no realistic initial change resp. reduction can be specified.

The RM Scenario Types are used in our tools to derive plausible global or national paths. The Excel tools can be downloaded from our [website](http://www.save-the-climate.info). Under [eu.climate-calculator.info](http://eu.climate-calculator.info) you will find a web application that also uses the RM Scenario Types.

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<sup>1</sup> "Increase" refers to the absolute amount of the reduction rates.

## 2 Predefined values

$B$	budget for a certain period (budget period); here: 2020 - 2100
$E_{BY}$	emissions in the base year ( $BY$ ); here: $BY = 2019$
$E_{min}$	minimum of emissions in the budget period; a negative value represents the potential for net negative emissions
$RR_{BY+1}$	rate of change in the first year of the budget period in RM 2 - 5; first year here: 2020; in scenario type RM-2, only a negative value is possible
$TV$	threshold from which the method is changed in order to be able to map net negative emissions (from this value a constant annual reduction amount is used)

## 3 Formulae Regensburg Model Scenario Types

### 3.1 Determination of paths via annual rates of change (scenario types RM 1 – 5)

$$E_t = \begin{cases} \max(E_{min}; E_{t-1} * (1 + RR_t)) & \text{for } E_{t-1} > TV^2 \\ \max(E_{min}; E_{t-1} + (E_{t-1} - E_{t-2})) & \text{for } E_{t-1} \leq TV^3 \end{cases}$$

where:

$E_t$  emissions in the year  $t$ ; here: 2020 – 2100

The **reduction rates** in the individual scenario types are based on the following formulae:

name scenario type	formula	basic function type	constraint	property course of the reduction rates
RM-2 exp <sup>4</sup>	$RR_t = RR_{t-1} * (1 + a)$	$e^x$	$a \geq 0$	► concave
RM-4 quadr <sup>5</sup>	$RR_t = a * (t - (BY + 1))^2 + RR_{BY+1}$	$y = ax^2 + b$	$a \leq 0$	
RM-5 rad <sup>6</sup>	$RR_t = a * \sqrt{t - (BY + 1) - 0.5} + RR_{BY+1}$	$y = a\sqrt{x} + b$	$a \leq 0$	► convex
RM-3 lin	$RR_t = a * (t - (BY + 1)) + RR_{BY+1} = RR_{t-1} + a$	$y = ax + b$	$a \leq 0$	► linear
RM-1 const	$RR_t = a$	$y = a$	$a \leq 0$	► constant

In the scenario types RM 2, 5 and 3 for  $t = BY+1$  the predefined  $RR_{BY+1}$  (see chapter 2) must be used. Thus the equations above hold for  $t > BY+1$  (here:  $t > 2020$ ).

The free parameter  $a$  is determined for each scenario type using an iterative solution method so that the budget ( $B$ ) is adhered to. In the Excel tools, the integrated target value search (“goal seek”) is used for this purpose, which is embedded in a macro that ensures that the constraint for  $a$  is also met (see program code in the attachment).

<sup>2</sup> "Max" means here, take the larger value. Either  $E_{min}$  or  $E_t$ , which results from the application of  $RR_t$ .

<sup>3</sup> "Max" means here, take the larger value. Either  $E_{min}$  or  $E_t$ , which results from the application from the last absolute reduction amount (the emission path is then a straight line).

<sup>4</sup> In this scenario type, the free parameter  $a$  can be called the escalation rate applied to the reduction rate of the previous year. This scenario type can also be represented using the following formula:  $RR_{BY+1} * e^{(t-(BY+1))*\ln(1+a)}$ .

<sup>5</sup> Basic function type:  $y = ax^2 + b$ . The term  $[t - (BY + 1)]$  is set for  $x$  in a variable transformation in order to be able to calculate with years. For  $t = 2020$  the value of the term is 0. The term thus takes the values 0, 1, ..., 80 for the period 2020 - 2100 considered here.

<sup>6</sup> Basic function type:  $y = a\sqrt{x} + b$ . The term  $[t - (BY + 1) - 0.5]$  is set for  $x$ . 0.5 serves to smooth the course at the beginning (see attachment). The term  $[t - (BY + 1)]$  represents a variable transformation in order to be able to calculate with years.  $x$  thus takes the values 0.5, 1.5, ..., 79.5 in the period 2021 - 2100 considered here.

### 3.2 Determination of paths via annual change amount (scenario type RM-6)

$$\text{RM-6 abs: } E_t = \max(E_{min}; E_{t-1} + RA)^7$$

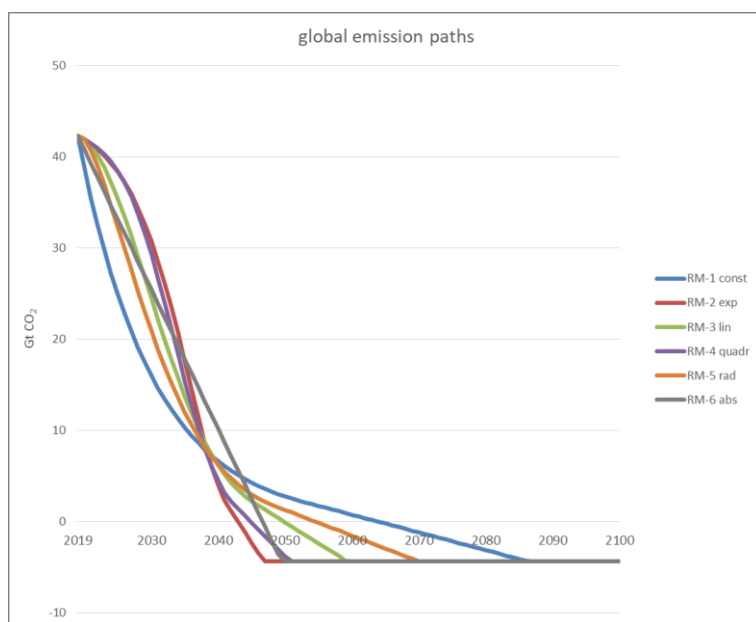
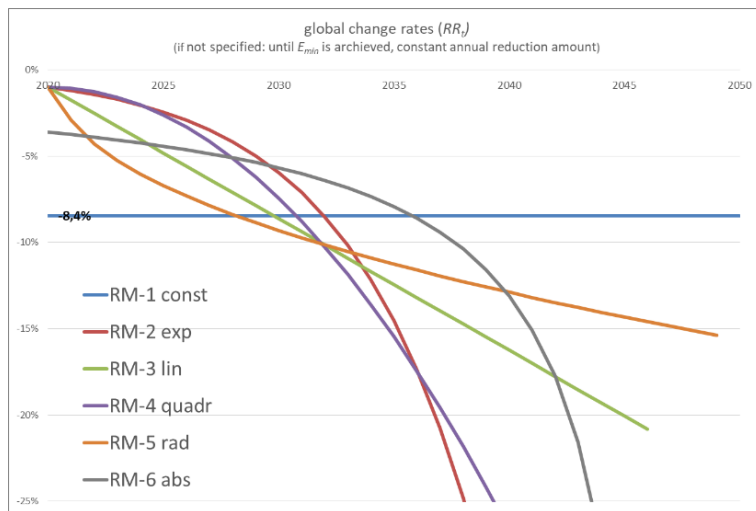
The free parameter  $RA$  (constant annual reduction amount) is determined using an iterative solution method so that the budget ( $B$ ) is adhered to. In the Excel tools, the integrated target value search (“goal seek”) is used for this purpose.

### 4 Phases for determining the paths

This usually leads to the following **three phases** for determining the paths:

1. Application of the annual **reduction rates** (RM 1 - 5) or **reduction amount** (RM-6).
2. RM 1 – 5;  $E_{t-1} \leq TV$ : The absolute reduction amount in  $t - 1$  is used from now on until  $E_{min}$  is reached. In this phase, the emission path is a straight line (as with RM-6).
3. Minimum for the annual emissions ( $E_{min}$ ) is used until 2100.

### 5 Exemplary global reduction rates and paths RM 1 – 6



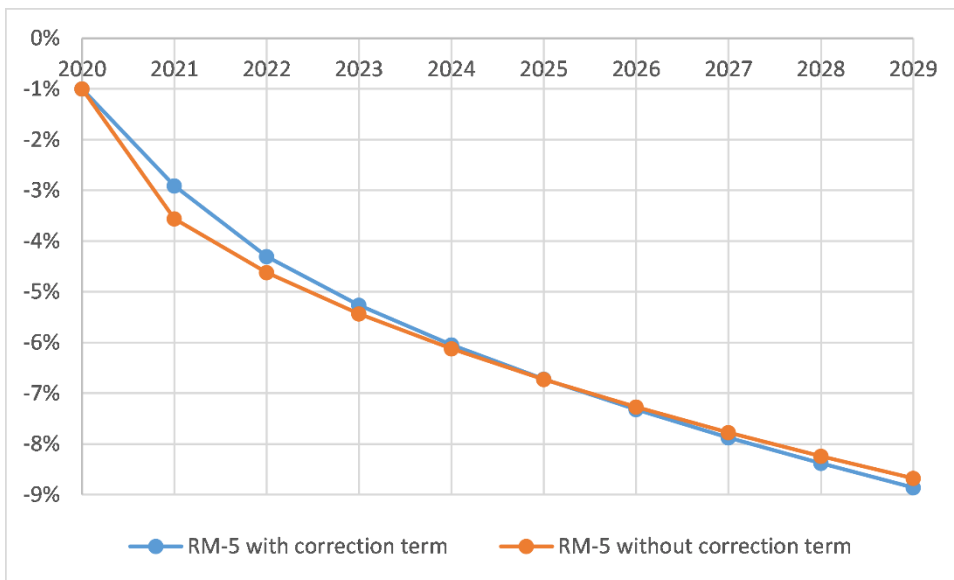
<sup>7</sup> "Max" means here, take the larger value. Either  $E_{min}$  or  $E_t$ , which results from the application of the constant annual reduction amount ( $RA$ ).

## 6 Attachment

### 6.1 Correction term RM-5

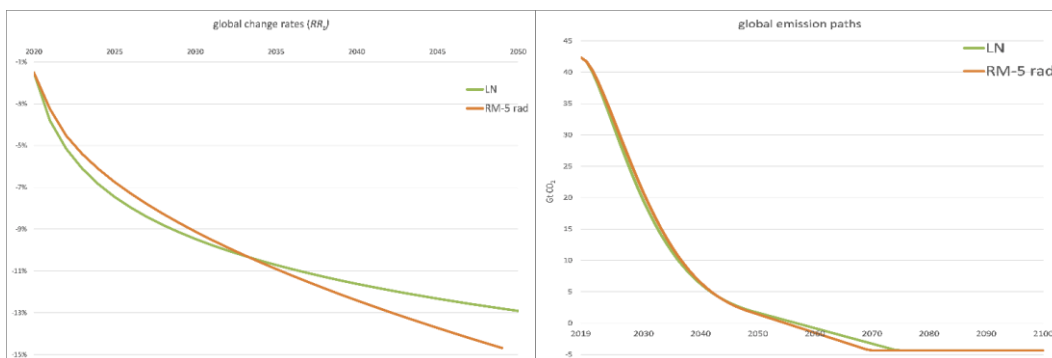
$$RR_t = a * \sqrt{t - (BY + 1) - 0.5} + RR_{BY+1}$$

As shown in the chart below the interaction of the weighting factor  $a$  and the root without the correction term  $0.5$  in RM-5 rad would result in a relatively large step in the reduction rates from the first year in the budget period (2020;  $BY+1$ ) to the second year (2021). With the correction term of  $0.5$ , this curve is "smoothed".



### 6.2 Further possible scenario types

- Concave:  $RR_t = RR_{BY+1} * e^{a*(t-(BY+1))}$   
This variant is almost congruent with RM-2 exp in the area used here.
- Convex:  $RR_t = a * \ln(t - BY) + RR_{BY+1}$   
The following graphics show the difference to RM-5 rad:



- Of course, a function for the emission path can also be specified directly. In the RM Scenario Types, however, the focus is on the property of the annual changes. The focus on the necessary annual reduction rates makes the challenge clearer and makes it easier to choose a meaningful emission path.

### 6.3 Extracts from the program code of the macros in the Excel tools

Cell references:

- row 9: change rate 2020 ( $RR_{BY+1}$ )
- row 12: free parameter  $a$
- row 15: sum of emissions 2020 - 2100 in the scenario using the formulas shown above
- row 16: given budget ( $B$ )

Code (you can request the complete code from us as a bas file):

'target value search: determination of the free parameter

```
Range("E15").GoalSeek goal:=Range("E16"), ChangingCell:=Range("E12")
```

'Try to find a solution with a changed rate of change for 2020 (E9) if the free parameter is not  $\leq 0$

```
adaption = -0.0055
```

```
step = 0.0005
```

```
number = 70
```

```
initial_value = Range("E9")
```

```
Do While Range("E12") > 0
```

```
  i = i + 1
```

```
  Range("E12") = -0.01
```

```
  adaption = adaption + step
```

```
  Range("E9") = initial_value + adaption
```

```
  Range("E15").GoalSeek goal:=Range("E16"), ChangingCell:=Range("E12")
```

```
  If i > number And Range("E12") > 0 Then
```

```
    MsgBox "The macro did not find a solution in scenario 3. Please choose a significant different RR_20 and restart the macro 'Goal Seek'."
```

```
    Range("E9") = initial_value
```

```
    i = 0
```

```
    GoTo b
```

```
  End If
```

```
Loop
```