working paper

Where to Go To with CO_2 Emissions

A Comparison of Resource Sharing Models

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Abstract

The COP 21 in Paris decided to start a ratchet up mechanism in 2018 that leads to progressive NDCs. If the connection between the cumulative CO_2 emissions and global warming, as described in the IPCC Fifth Assessment Report, is taken seriously, the question arises whether the progressive NDCs submitted by the countries allocate this remaining CO_2 budget in a fair and reasonable manner.

Resource sharing models directly address the allocation of such a remaining global budget. This article will therefore give an overview of the properties of selected resource sharing models.

We found out a new essential property of convergence models: convergence models show under certain conditions a specific implicit weighting of population depending of the selected global pathway. This implicit weighting is obtained by allocation a global remaining budget directly to countries using a weighted distribution key of “current emissions” and “current population” such that the same national budgets are obtained as in the convergence model.

Against this background the question arises, which implicit weighting of population contain NDCs. The implicit weighting of population could be an important additional key indicator, when it comes to a rational discourse on realistic and targeted NDCs.

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Contents

1 Introduction and underlying data

According to the Paris Agreement, countries must regularly present their ambitions (NDCs), i. e. their plans for CO_2 emissions. This leads to the question, what criteria their ambitions are based on.

The framework is given by the IPCC, which considers a cumulative budget of 2,900 $GtCO_2$ since the beginning of industrialisation to be compatible with the 2 °C limit with a probability of over 66%. About 1,890 $GtCO_2$ of this budget were already emitted by the end of 2011 (cf. IPCC, 2013, p. 27; IPCC, 2014, p. 24). Hence the remaining cumulative budget from 2011, i. e. as of 2012, is approximately 1,000 $GtCO_2$ with current global annual anthropogenic emissions of about 41 $GtCO_2$ (cf. Le Quéré, et al., 2017).

In this paper we have concentrated on resource sharing models[1] currently discussed, which take into account current emissions and population when it comes to determining national emission pathways. We selected these models, as they are suitable for indicating realistic NDCs, which in sum fulfil the 2° C target. Current emissions reflect the present reality and population can map justice. This article wants to present different and common features and increase the transparency of the models.

In Chapter 2 we consider models with a limited convergence period, at the end of which global emissions are allocated to countries according to population only. The Smooth Pathway Model in Chapter 3 calculates national pathways starting from allocated remaining national budgets. The Emission Probability Model in Chapter 4 determines country specific emission density functions and caps the emissions of individuals.[2]

By way of illustration we show the results of the models for three pure type countries (see Chapter 2.4 and Chapter 5) with the following underlying data:

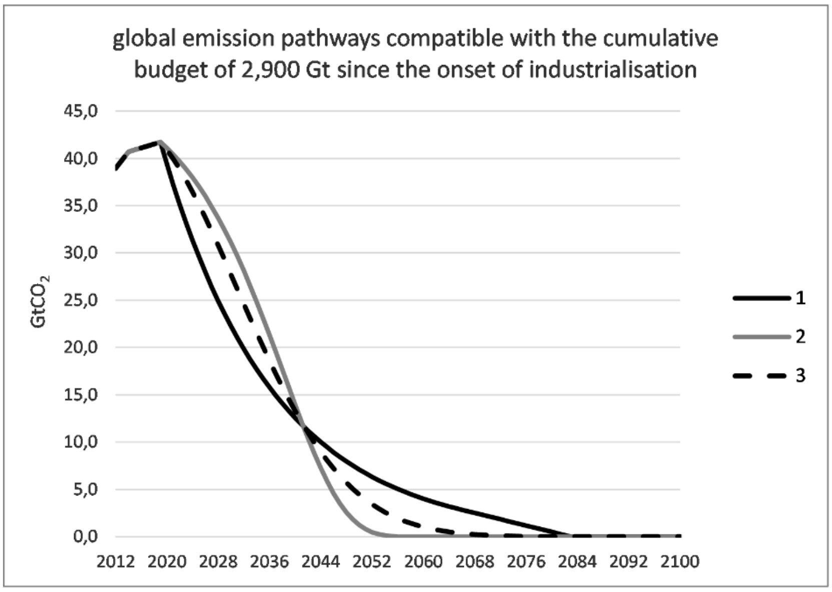

Figure 1: Global emission pathways. Details can be found in the Supplementary Tool 2.

in the base year 2019	country			world
	A	B	C	
	industrial	emerging	developing	
emissions in $GtCO_2$	21.0	19.5	1.2	41.7
population in billions	1.2	4.0	1.7	6.9
per capita emissions in t	17.5	4.9	0.7	6.1

Table 1: Underlying data for the three pure type countries. Details including the values of further parameters can be found in the Supplementary Tool 1.

Our aim is not to explore real countries but pure type countries of which per capita emissions are typical of an industrial, an emerging and a developing country. In sum the pure type countries approximately reflect the global data. In our calculations we used a “frozen” population of the base year.

For a mathematical description of the models with proofs of its properties we refer to the Supplementary Text.

2 Convergence models

All convergence models presented here start with a global pathway that meets a remaining global budget usually corresponding to a certain degree of global warming. Then the models break down the annual global emissions on country level, transforming the actual emissions in a base year (*BY*) into emissions based on a per capita allocation in a convergence year (*CY*) at the end of a limited convergence period. In the illustration with the three pure type countries, we have chosen 2050 as the convergence year and 2019 as the base year.

The models in Chapters 2.1 and 2.2 gradually replace the allocation key "emissions in a base year" with the allocation key "population" within the convergence period. However, the underlying formulae are different in each model. In Chapter 2.3 we present enhancements of the previous models that enable different rules for some countries and in Chapter 2.4 we compare the models in Chapter 2.

2.1 Contraction & Convergence Model

The Global Commons Institute already propounded the following Contraction & Convergence Model (C&C) in the early 1990s. This model defines the emissions of country *i* in the year *t* $\left(\widehat{E_t^i}\right)$ recursively (Meyer, No date)[3]:

$$\widehat{E_t^i} := \begin{cases} \left((1-\widehat{C_t}) * \frac{\widehat{E_{t-1}^i}}{E_{t-1}} + \widehat{C_t} * \frac{P_t^i}{P_t}\right) * E_t, \text{ for } BY+1 \le t < CY \\ \frac{P_t^i}{P_t} * E_t, \text{ for } CY \le t \end{cases}, \tag{1}$$

where

E_t global emissions in the year *t*,

P_t global population in the year *t* and

P_t^i population of country *i* in the year *t*.

$\widehat{C_t}$ denotes the weighting of the population when allocating global emissions to countries.

The Global Commons Institute considered two specifications of $\widehat{C_t}$:

- exponential (C&C-exp): $\widehat{C_t} = exp\left(-a\left(1 - \frac{t-BY}{CY-BY}\right)\right)$ with the parameter $a > 0$ to be determined. "*The higher the value [a], the more the convergence happens towards the end of the convergence period, and vice-versa. Choosing a = 4 gives an even balance.*" (Meyer, 1998, p. 21)

- linear (C&C-lin): $\widehat{C}_t = \frac{t - BY}{CY - BY}$.

Some more specifications of $\widehat{C}_t$ are presented and discussed in the Supplementary Text.

2.2 The Regensburg Model

In the Regensburg Model (RM) the emissions of country *i* in the year *t* $\left(\overline{E_t^i}\right)$ are given by

$$\overline{E_t^i} := \begin{cases} (1 - \overline{C}_t) * E_{BY}^i + \overline{C}_t * E_{CY}^i, \text{ for } BY + 1 \leq t < CY \\ \frac{P_t^i}{P_t} * E_t, \text{ for } CY \leq t \end{cases} \quad (2)$$

where $\overline{C}_t = \frac{E_{BY} - E_t}{E_{BY} - E_{CY}}$ and $E_{CY}^i = \frac{E_{CY}}{P_{CY}} * P_{CY}^i$ (cf. Sargl, et al., 2017).

2.3 Different pathways for emerging and developing countries

In this chapter we present enhancements of the previous models that enable more favourable rules for some (normally emerging) countries. These modifications involve greater efforts to be made by the other (normally industrial) countries.

2.3.1 Common but Differentiated Convergence Model

The Common but Differentiated Convergence Model (CDC) refines C&C (cf. Höhne, et al., 2006). *"This approach [CDC] eliminates two concerns often voiced in relation to gradually converging per-capita emissions: (i) advanced developing countries have their commitment to reduce emissions delayed […] (ii) CDC does not provide excess emission allowances to the least developing countries."* (Höhne, et al., 2006, p. 181) This is achieved by allocating countries below a continuously decreasing threshold emissions according to their free decision recorded in a business-as-usual scenario. Thus the C&C model is only used for countries with per capita emissions above this threshold.

2.3.2 Modified Regensburg Model

The RM can also be combined with the idea of CDC, where some countries are exempt from the emission allocation regime as long as their per capita emissions are below the threshold.

It is even possible to exempt some countries from the emission allocation regime throughout the convergence period. This would be a way of allowing, for example, countries which start significantly below the convergence level in the base year to get emissions according to straight pathways to the convergence level. This “shortest way to the convergence level” can be seen as a minimum justice level for developing countries.

In this case “global”, in the description of formula (2), must be read as “of the countries under the emission allocation regime”.

2.4 Comparison of the convergence models

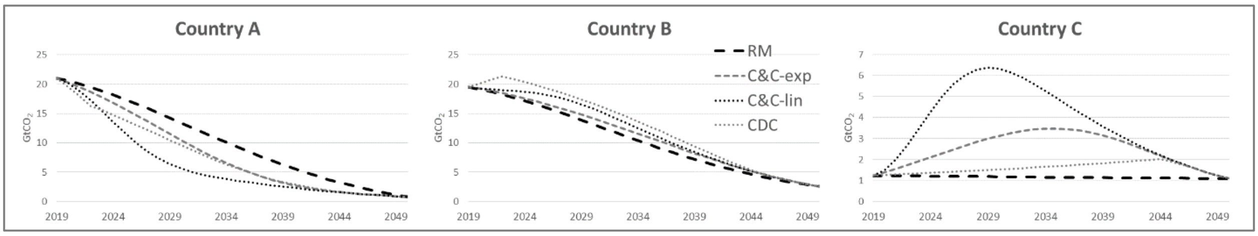

Figure 2: Comparison of the convergence models – emission pathways (in the C&C-exp model we chose a = 4).

Figure 2 shows that the RM requiring the least reductions for country A (typical industrial country) corresponds to greater efforts for country B (typical emerging country) and country C (typical developing country).

If the effects of changes in population are suppressed and if global emissions fall during the convergence period, then the RM in contrast to the other models allocates

- countries starting above the convergence level lower emissions from the first year on in the convergence period, irrespective of how far and how long they have already been above the convergence level. As a consequence, most emerging countries have to reduce their emissions from the first year on in the convergence period.
- countries starting below the convergence level higher emissions in each year, but, in contrast to other models in Chapter 2, never greater emissions than the convergence level. The RM, therefore, similarly to CDC, does not provide developing countries with excess emission allowances.

For a given global pathway the weight of population $\widehat{C}_t$ in C&C can be calculated in such a way that the resulting national pathways of C&C and the RM are the same if the population is frozen.

This allows making clear the different weighting of population in the RM and in C&C-exp. In the following illustrating figure we used the global pathway number 3 in Figure 1.

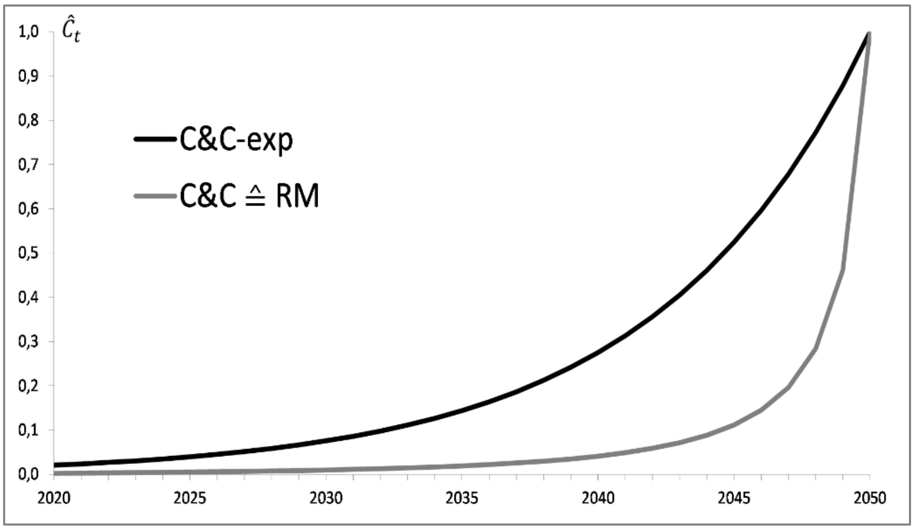

Figure 3: Comparison of the weighting of population in C&C-exp (a=4) and in C&C yielding the same results as the RM.

We also examined whether the results of the RM and the C&C model with the "classic exponential" specification of C&C (C&C-exp) are similar. Choosing the parameter a = 8, we obtained similar results for the industrial country A and the emerging country B. However, the results for the developing country C are different and not plausible: a sharp increase of emissions at the end of the convergence period, i. e. shortly before the intended decarbonisation, is followed by a sharp drop in emissions.

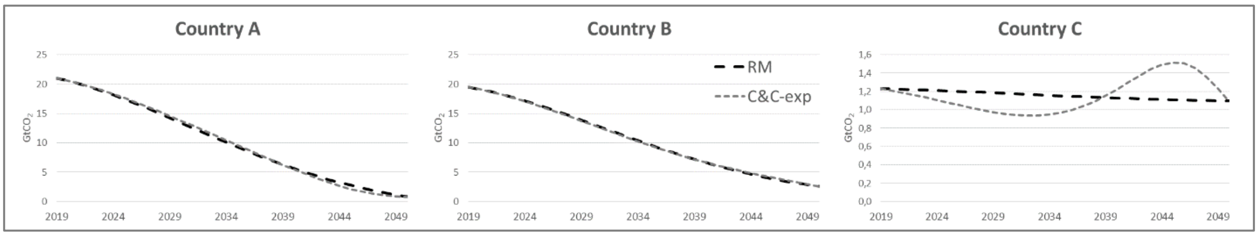

Figure 4: Comparison of the RM and C&C-exp (a = 8) - emission pathways

We would like to stress that in the RM $\overline{C_t}$ is determined by the underlying global pathway. In C&C, by contrast, $\widehat{C_t}$, which specifies how fast the per capita distribution comes into effect, can be chosen arbitrarily.

3 Smooth Pathway Model

Raupach, M. R. *et al.* suggest the following weighted key for allocation the remaining global budget to countries (cf. Raupach, et al., 2014)):

$$RB^i = \left(\check{C}_{RB} * \frac{P_{BY}^i}{P_{BY}} + \left(1 - \check{C}_{RB}\right) * \frac{E_{BY}^i}{E_{BY}}\right) * RB \qquad (3)$$

where

RB global remaining budget

RB^i remaining budget of country i

$\bar{\bar{C}}_{RB}$ weighting of population

A lot of criteria on how to obtain a remaining budget for each country (gross domestic product (per capita), cumulative emissions per capita, population, emissions in the past, …) are possible.

But there are two outstanding criteria: population in a base year (equity) and emissions in a base year (inertia). *"These two alternatives act as bounds to a range of blended options, and demonstrate how national quotas [(remaining national budgets)] can be allotted using any mix of the two alternatives, [...]."* (Peters, et al., 2015, p. 3).

Raupach, M. R. *et al.* also showed how to transform the allocated remaining budget of a country into a positive pathway (i. e. a pathway which has no negative emissions), with a smooth transition from the current pathway and with near-zero emissions at infinity.

In the Smooth Pathway Model (SPM) the emissions of country *i* in the year *t* (E_t^i) are given by

$$E_t^i = -\dot{E}_{BY+1}^i \frac{e^{-m^i(t-BY)}}{(m^i)^2}\left[\left(r^i m^i + \left(m^i\right)^2\right)(t-BY) + 2m^i + r^i\right]$$

$$+\dot{E}_{BY+1}^i \frac{e^{-m^i(t-BY-1)}}{(m^i)^2}\left[\left(r^i m^i + \left(m^i\right)^2\right)(t-BY-1) + 2m^i + r^i\right],$$

where

$\dot{E}_{BY+1}^i$ emission power, i. e. the derivative of emissions with respect to time or the emissions per unit of time, of country *i* at the end of the base year,

r^i change rate of the emission power of country *i* at the end of the base year and

m^i the mitigation rate (or the decay parameter) of country *i*.

If $r^i > -1/T^i$, the mitigation rate m^i is given by

$$m^i = \frac{1 + \sqrt{1 + r^i T^i}}{T^i},$$

where $T^i = \frac{RB^i}{\dot{E}^i_{BY+1}}$ is the time defined by the remaining budget of country *i* and the emission power of country *i* at the end of the base year.

4 Emission Probability Model

The Emission Probability Model (EPM) from Chakravarty, S. *et al.* not only takes emissions and population into consideration, but also the income distribution of a country (cf. Chakravarty, et al., 2009). EPM assumes that the emission distribution is a scaled income distribution. Then EPM allocates a country the emissions of its inhabitants whose emissions are below a cap and the cap for each inhabitant whose emissions are above the cap. The global cap is chosen each year such that the global emissions are met. Thus EPM, as well as the convergence models in Chapter 2, allocates a preset global pathway to all countries.

In EPM the emissions of country *i* in the year *t* are given by

$$E_t^i = P_t^i \left(\int_{-\infty}^{CA_t} z \, \tilde{f}^i(z; p^i) \, dz \; + CA_t \int_{CA_t}^{\infty} \tilde{f}^i(z; p^i) \, dz \right), \tag{4}$$

where

CA_t the cap in the year t and

$\tilde{f}^i(z; p^i)$ the estimated emission probability density function (PDF) of country *i* with parameters p^i.

5 Comparison of the models

The following two figures show the emission pathways and the per-capita emissions of the pure type countries resulting from the presented models for a comparison:

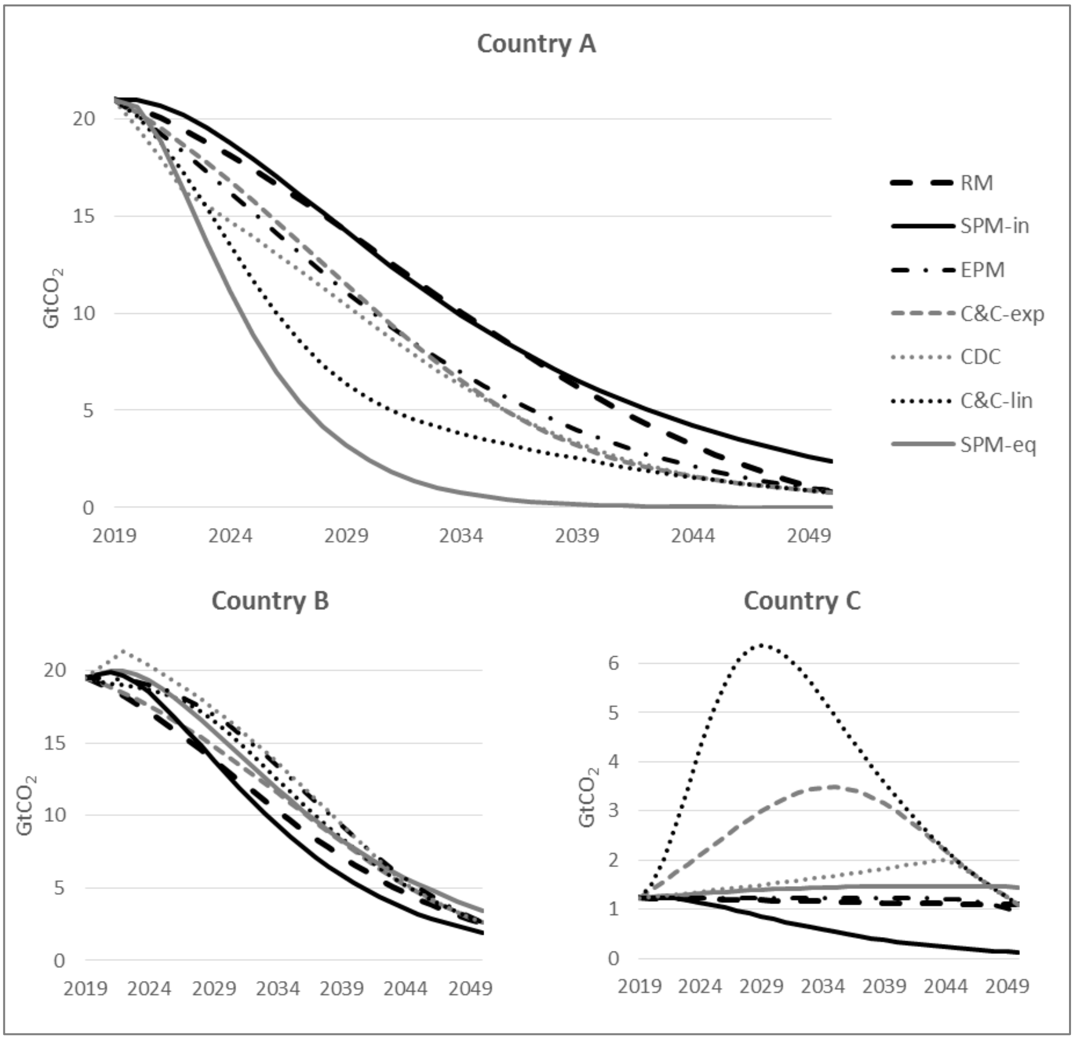

Figure 5: Comparison of Resource Sharing Models – emission pathways

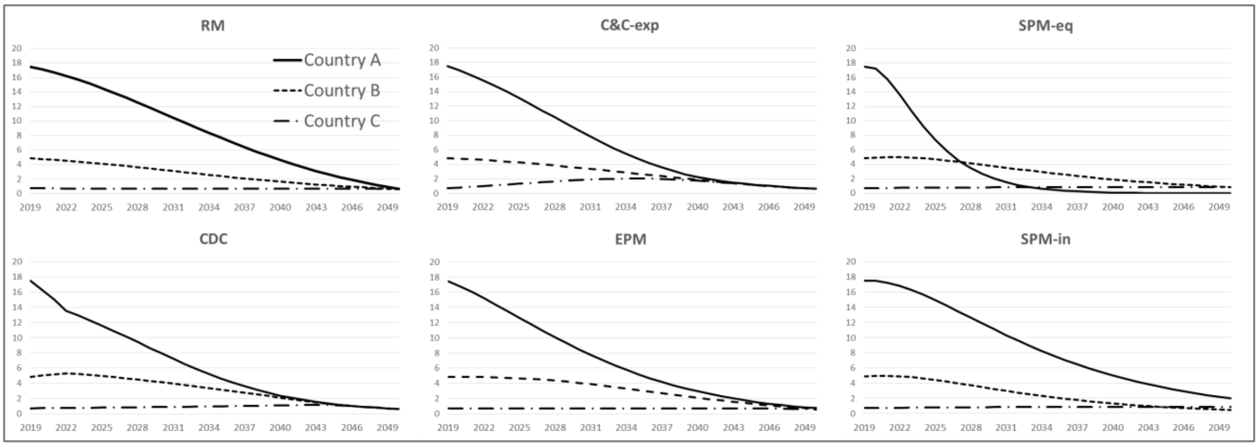

Figure 6: *Comparison of Resource Sharing Models – t per capita emissions*

Figure 5 shows that, if SPM-in is not taken into account, RM is clearly the most favourable resource sharing model for industrial countries. For a detailed comparison of the models in Chapter 2 we refer to the results in Chapter 2.4.

The following key figures of the models can be reproduced with the help of the Supplementary Tool 1.

Approximation of C&C, RM and EPM with SPM

In the model with three pure type countries and the underlying global pathway (number 3 in Figure 1) we approximated the pathways of C&C-exp (a = 4), RM and EPM with SPM, minimising the sum of the squared relative deviations in each year . By this means we obtained a weighting of the population (see Formula (3)) of 50% for C&C-exp (a = 4), 16% for RM (see Figure 7) and 24% for EPM.

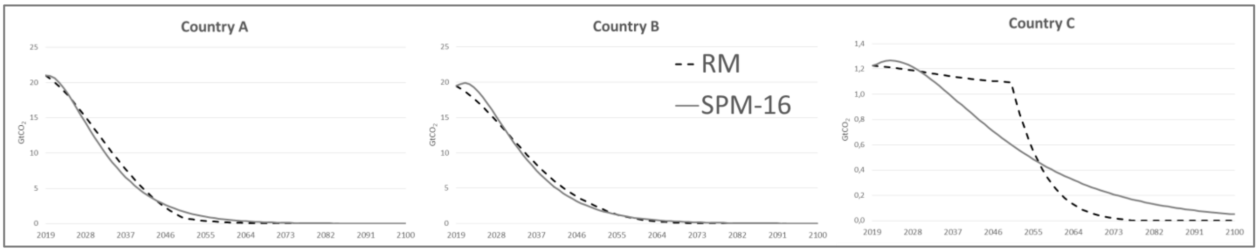

Figure 7: Comparison of SPM (weighting of population = 16%) and RM - emission pathways

Consideration of population

The idea of the models in Chapter 2 is an annual increase of the influence of the allocation key “population”, whereas the SPM can only consider population once when determining national budgets. EPM does not take into account population as an allocation key, but calculates a global cap for per capita emissions.

Convergence

Convergence models lead to positive identical per capita emissions in a given convergence year. The SPM leads to zero emissions at infinity in each country. Therefore, from a mathematical point of view, SPM leads to per capita emissions that are converging to zero. EPM only leads to converging per capita emissions when global emissions are tending to zero.

Role of the global pathway

In contrast to the other resource sharing models, SPM does not share a global pathway, but a remaining global budget. National positive pathways can then be calculated and the global pathway is obtained by summing up national pathways. The global pathway is therefore an output value of SPM, whereas in the other models the global pathway is an input value (the global pathway in SPM-in meets the global cumulative budget of 2,900 $GtCO_2$ up to and including the year 2100; the global pathway in SPM-eq even meets a cumulative budget of 2,820 $GtCO_2$ up to and including the year 2100).

Dependence of remaining national budgets on the global pathway

In the convergence models in Chapter 2 and EPM (Chapter 4) the remaining national budgets also depend on the choice of the global pathway. Here the principle holds that global pathways that stipulate high reductions only at a late stage are more favourable - from the perspective that the reduction of emissions carries disadvantages for a country - for industrial countries than global pathways that stipulate high reductions at an early stage. This property holds all the more for the RM, since its weighting of population also depends on the global pathway.

In order to illustrate the impact of the choice of the global pathway, we considered significant different positive global pathways (see Figure 1) meeting the global cumulative budget of 2,900 Gt CO_2. We then calculated with the different global pathways the remaining national budgets that result from the RM, C&C-exp, C&C-lin and EPM. We also calculated the remaining national budgets directly with a weighting of the allocation keys “population” and “emissions in a base year” using Formula (3). We then minimised over the weighting of population the sum over each country of the squared relative deviations of the two remaining national budgets (best approximation of the remaining national budgets of the RM, C&C and EPM with a direct blended allocation of population and emissions in a base year). This led to the results in Table 2.

Number of the global pathway in Figure 1	2	3	1
RM – pure type country model and real world	3%	15%	35%
C&C-exp – pure type country model and real world	39%	43%	51%
C&C-lin – pure type country model and real world	68%	69%	73%
EPM – pure type country model	11%	18%	30%

Table 2: Weighting of population resulting from different global pathways

If the population is frozen and if there are no global negative emissions in the period under consideration the results of the convergence models (RM, C&C-exp and C&C-lin) do not depend on whether a „pure type country model“ or the “real world” is used. The weighting of population is hence a characterising key figure of the convergence model that depends only on the global pathway.

This finding leads to the idea that such a key figure should also exist for NDCs. It should be possible to deduct a national emission pathway and then the remaining national budget. Using this remaining national budget as well as the emissions and the population in a base year the implicit weighting of population can be calculated.

Further properties of the models under consideration

- In SPM with a low weighting of the population, developing countries have to reduce their emissions relatively quickly, but emissions remain at a higher level longer than in the RM (see Figure 7).
- If the effects of changes in population are suppressed and if global emissions fall, then the EPM is the only model that also continually reduces the emissions in the developing countries after a base year.
- SPM, CDC and EPM in one way or another take into consideration the change rate of emissions from the base year. Usually this leads to a soft transition from the emissions in the base year.
- SPM always leads to positive national pathways. Hence, the resulting global pathway is also always positive. SPM can therefore map neither global nor national negative emissions in a year.
- National pathways under the EPM will always decline. National pathways under the SPM will fall after they have reached a maximum. In contrast national pathways, particularly of developing countries in C&C, show a change of direction within the convergence period. The pathways of developing countries in the RM usually show a clear kink at the end of the convergence year, if the global pathway declines rapidly after the convergence period (see Figure 7).

6 Conclusion

The direct comparison of resource sharing models using three pure type countries has revealed significant differences between the models that are relevant when discussing reference values for the NDCs in the ratchet up mechanism of Paris, which will start with a facilitative dialogue in 2018.

If SPM-in is not taken into account, RM is the most favourable resource sharing model for industrial countries. Therefore national emission pathways calculated with the RM describe a floor of ambition for industrial countries if they accept taking into account equal per capita rights as an aim. They will otherwise have difficulty explaining their NDCs if they fall below this floor. As an example the RM means for the EU that it has to reduce emissions in comparison to 1990 rather in a scale of 50% than only 40% proposed so far (cf. Sargl, et al., 2017, p. 672).

In an overall assessment of the characteristics of the models, SPM has the advantage that the question of climate justice is addressed explicitly and does not depend on a global pathway. Furthermore SPM leads to smooth pathways until infinity. Other models might give hints of the range of the weighting of population when it comes to determining remaining national budgets as an input value in SPM.

If any other model is used to justify or to assess NDCs, the underlying global pathway must be disclosed, because its results depend on the selected global pathway.

The question of the convergence of the per capita emissions as a property of a model becomes less important the sooner global emissions must be zero or even negative. The later global emissions begin to decline, the sooner emission neutrality must be reached. This is due to the budget property of CO_2.

Although in SPM it is not necessary to determine a global pathway when allocating the global remaining budget to countries, it would be useful for the world community to agree on a suitable global pathway, after having agreed on a remaining global CO_2 budget compatible with the 2 °C limit. This would enable an effective control of deviations. The global pathway would not be carved in stone, but could and should be constantly adapted to the latest developments and scientific status.

Considering the ratchet up mechanism of Paris the focus should be more on the remaining national cumulative budgets seen as a fair and reasonable share of the remaining global cumulative budget. We have shown: resource sharing models give useful help when it comes to determining these budgets.

It would make sense to calculate the weighting of the allocation keys “current population” and “current emissions” leading to the same remaining national budgets that result from NDCs. This makes clear, which implicit weighting of the allocation key “current population” is considered as legitimate. Thus, this weighting of population could contribute to a more rational discourse of the core question: Who gets how much of the remaining global cumulative budget?

7 Notes

[1] “There are two broad approaches to sharing emissions reduction efforts:

- sharing the global emissions budget - ‘resource-sharing’
- sharing the emissions reductions required to meet that budget - ‘effort-sharing’ [or ‘burden sharing’] […].

In some ways, the two approaches are similar - sharing the remaining budget implicitly sets a mitigation task and vice versa. From a practical perspective, resource-sharing approaches are more straightforward, as they require only an estimate of the global emissions budget and equitable principles. In contrast, effort-sharing also requires an estimate of global emissions in the absence of climate change action; that is, a BAU trajectory. As more countries take more action, this trajectory becomes increasingly abstract and difficult to estimate.” (Australian Government Climate Change Athority, No date)

[2] Du Pont, Y. R. *et al.* distinguish five allocation approaches: (1) capability, (2) equality, (3) responsibility, (4) equal cumulative per capita and (5) staged approaches (du Ponte, et al., 2017, p. 40). Using this classification, the models in Chapter 2 belong to category (2). The classification of the Smooth Pathway Model in Chapter 3 depends on the determination of the remaining national budgets. The Emission Probability Model in Chapter 4 belongs rather to category (1) with “equality” from category (2) playing a role, too.

[3] LIMITS, a research project funded by the EU, defines the emissions of country *i* in the year *t* $\left(\widetilde{E_t^i}\right)$ explicitly (cf. Tavoni, et al., 2013):

$$\widetilde{E_t^i} := \begin{cases} \left(\left(1-\widetilde{C_t}\right) * \frac{E_{BY}^i}{E_{BY}} + \widetilde{C_t} * \frac{P_t^i}{P_t}\right) * E_t, & \text{for } BY + 1 \leq t < CY \\ \frac{P_t^i}{P_t} * E_t, & \text{for } CY \leq t \end{cases}$$

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9 Supplementary Material

Supplementary Material can be downloaded from www.save-the-climate.info (menu: "Downloads").

Supplementary Text: Resource Sharing Models - A mathematical description

Supplementary Tool 1 (Excel tool): Comparing Resource Sharing Models

Supplementary Tool 2 (Excel tool): Smooth Global Pathways (Tool_global_paths.xlsm)